Short-term Scheduling of Batch Plants with Multiple Intermediate Due Dates

M. G. Ierapetritou, T. S. Hené and C.A. Floudas [1]

Department of Chemical Engineering, Princeton University Princeton, N.J. 08544

Abstract
The problem of short-term scheduling often involves the satisfaction of variable product demands at specific due dates within the time horizon under consideration. Ierapetritou and Floudas, [2, 3] presented a novel continuous time formulation to effectively address the problem of short-term scheduling in batch, continuous and mixed production facilities where product demands are specified at the end of time horizon. The primary objective of this paper is to extend the continuous time formulation so as to deal with intermediate due dates. The mathematical model is developed and the features of the problem are further exploited to result in the most efficient solution of the problem. Two examples are provided to illustrate the capabilities of the proposed approach.

Introduction
There has been a considerable amount of work in the literature for the problem of short-term scheduling of both batch and semi-continuous plants involving intermediate due dates. Among them are the works of Sahinidis and Grossmann [8] presenting a multiperiod MILP model for batch plants; Mockus and Reklaitis [5] proposing a nonconvex Mixed Integer Nonlinear Programming (MINLP) model and global optimization algorithm for its solution for the case of batch plants; Pinto and Grossmann [6] with a MILP formulation for batch plants without resource constraints; and Karimi and McDonald, [1] proposing a MILP formulation a single-stage multiproduct semi-continuous plant.
In this work, the objective is to effectively address the requirement of multiple intermediate due dates in the short-term scheduling problem for batch plants. Two models are proposed based on the basic concepts of the approach of Ierapetritou and Floudas, [2, 3]. The first model facilitates the solution of the problem where no resource constraints are considered and the second model considers the overall short-term scheduling problem of batch plants including resource constraints.

Basic Concepts of Continuous Time Formulation
The proposed formulation for short-term scheduling of batch plants with multiple due dates is based on the following novel concepts of the framework presented in Ierapetritou and Floudas [2, 3]: (a) it follows a *continuous time* representation in which the event points, where a task starts being processed and/or a unit starts operation, are unknown and constitute variables to the optimization problem; (b) it uses different binary variables for *task events*, $wv(i,n)$, that represent the start of task (i) at event point (n), and different binary variables for *unit events*, $yv(j,n)$, that correspond to the beginning of unit (j) utilization at event point (n); (c) it allows for variable processing times with respect to the amount of material processed by the specific task.

No Resource Constraints
The proposed mathematical formulation for the short-term scheduling of batch plants with multiple intermediate due dates has the following characteristics:

Preordering: Orders are put in sequence a priori and linked to nodes.

Binary Variables: The binary variables $wv(i,n)$ employed in [2, 3], become parameters since each order (i) is linked to a particular event point (n), for which $wv(i,n)$ is set to 1. The $wv(i,n)$ are set to 0 for all other combinations. Consequently, $yv(j,n)$ are the only binary variables.

Elimination of Material Balance: The material balances are eliminated from the proposed formulation, since no resource constraints are considered.

Elimination of States: The states are superfluous due to the absence of resource constraints. Tasks are instead directly related to their due dates.

Due dates: Due dates are set to the end of the time intervals, since tasks performed in interval (n) are ready at the end of this node.

Mathematical Model
Using the aforementioned characteristics the mathematical formulation involves the following constraints:
Allocation Constraints

$$\sum_{j\in J_i} yv(j,n) = 1, \quad \forall i \in I, n \in N_i \qquad (1)$$

These constraints express that an order only has to be processed once, since only one unit can be assigned to it.
Duration Constraints

$$T^f(i,j,n) = T^s(i,j,n) + yv(j,n)(\alpha_{ij} + \beta_j) \quad \forall i \in I_j, j \in J_i, n \in N_i \qquad (2)$$

These constraints ensure that the processing time of an order (i) in a particular machine (j) is the sum of the setup time characteristic in the unit, (β_j), and the processing time of this order in this particular unit, (α_{ij}).
Sequence Constraints: Different tasks in the same unit

$$T^s(i',j,n_{i'}) \geq T^f(i,j,n_i) \quad \forall i \in I_j, i' \in I_j, j \in J_i, n_{i'} > n_i, i \neq i' \qquad (3)$$

[1] Author to whom all correspondence should be addressed; Tel: (609) 258-4595; Fax:(609) 258-0211;email:floudas@titan.princeton.edu

These constraints express the requirement that only one order at a time can be processed in a unit; the beginning of a later event must follow after the end of all earlier events in the same unit.

Due Dates Constraints

$$T^f(i,j,n_i) \leq due(i) \quad \forall i \in I, \ \forall j \in J_i \tag{4}$$

where n_i corresponds to the assignment of node (n) to order (i). These constraints ensure that the order would be ready at its due date, $due(i)$.

Objective function

Maximize

$$\sum_i wt(i) \sum_{j\in J_i, n\in N_i} [T^s(i,j,n) + yv(j,n)(\alpha_{ij} + \beta_j)] \tag{5}$$

The objective function represents the maximization of the weighted starting times for orders in all stages, and determines the schedule that satisfies the due dates in a most efficient way.

Illustration

An example from Pinto and Grossmann,[6], is considered here in order to illustrate the applicability of the proposed approach and its advantages compared to the published approaches.

The coupling of orders to time slots was performed according to the following rules, as suggested by Pinto and Grossmann, [6], (a) increasing due dates and (b) decreasing processing times (if ties occur). Based on these preordering rules, the proposed formulation is applied for the case where 29 orders are considered. The results of the proposed approach are presented in Table 1 and compared with the results of the formulation proposed by Pinto and Grossmann, [6]. Note that the proposed formulation results in smaller models in terms of binary and continuous variables and constraints. In particular, the proposed formulation requires 57 binary and 172 continuous variables, in comparison to 441 binary and 875 continuous variables required by the Pinto and Grossmann, [6], formulation. In addition, the proposed formulation requires 559 constraints instead of 1791 used by Pinto and Grossmann, [6]. Moreover, the resulting model is more easily solved requiring the exploitation of only 5 nodes in the branch and bound tree and the need of 0.28 CPU sec using GAMS/CPLEX on a HP C160 workstation. Figure 1 depicts the gantt chart of the optimal solution. The numbers below the horizontal lines indicate the order number being processed at that time.

Consideration of Resource Constraints

The overall short-term scheduling problem is considered here including resource constraints. The explicit consideration of resource constraints requires that we treat the $wv(i,n)$ as binary variables and not as parameters. Also, for the introduction of material balances we need to introduce the states (s) and their amounts at event point (n), $R(s,n)$. The mathematical model involves the following constraints:

Allocation Constraints

$$\sum_{i\in I_j} wv(i,n) = yv(j,n) \quad \forall j \in J, n \in N \tag{6}$$

These constraints ensure that only one task (i) can take place at unit (j) at each event point (n).

Capacity Constraints

$$V_{ij}^{min} wv(i,n) \leq B(i,j,n) \leq V_{ij}^{max} wv(i,n) \quad \forall i \in I, j \in J_i, n \in N \tag{7}$$

These constraints express the minimum and maximum capability of unit (j) when processing task (i), enforced at every event point (n).

Material Balance

$$\begin{aligned} d(s,n) &= \sum_{i\in I_s} \rho_{si}^p \sum_{j\in J_i} B(i,j,n-1) \\ &\quad + \sum_{i\in I_s} \rho_{si}^c \sum_{j\in J_i} B(i,j,n), \\ &\quad \forall s \in S, n \in N \end{aligned} \tag{8}$$

According to these constraints the amount of material of state (s) delivered at event point (n), $d(s,n)$, is equal to the amount produced between the event points $(n-1)$ and (n) denoted as $B(i,j,n-1)$, minus the amount consumed between event points (n) and $(n+1)$ denoted as $B(i,j,n)$.

Demand Constraints

$$d(s,n) = R(s,n), \quad n \in N, s \in S \tag{9}$$

These constraints ensure the satisfaction of product demand at event point (n), $R(s,n)$. Two important features of the model are the link of demands to event points and the due date constraints that ensure the satisfaction of product demand by the corresponding due date.

Linking Demands to Event Points

Demands have to be linked to particular event points, based on the relative time at which the demand has to be filled, the number of stages required to get to the final product, and the number of other tasks that may take place in the same unit. An important issue to note is that due to the nature of the batch operation mode the amount of state (s) produced in event point (n) cannot be consumed until the beginning of event point $(n+1)$. Therefore, even the most basic problem (one task with a single demand) requires two event points. In the first event point, a task produces the product, and at the beginning of the second event point it can meet the demand. Also, since the production time depends on the number of stages involved, the number of event points should be also stage dependent which means that if a task occurs in the second or higher stage, then additional event points need to be considered (e.g., one for stage two, two for stage three).

Due Dates Constraints

Following the aforementioned linking of demands to event points, the following set of time constraints ensure the satisfaction of product demand by the corresponding due date:

$$T^f(i,j,n_s) \leq due(s), \ \forall s \in S, i \in I_s, j \in J_i$$

Since the demand has to be met at the beginning of a particular node, the due date time limitations are incorporated as upper bounds on the final times of the tasks producing state (s). Finally, duration and sequence constraints are incorporated for tasks in the same or different units and in addition timing constraints for storage tasks, [2], [4]. The objective function corresponds to the minimization of the operating cost consisting of production cost, cost of raw materials and storage cost.

Illustration

In this section an example problem is presented to illustrate the applicability of the proposed formulation. The detailed data for the problem could be found at [8], and [5], batch 2 example. The STN representation of the plant flowsheet is shown in Figure 2. The resulting MILP formulation involves 441 constraints, 316 continuous variables and 54 binary variables. The solution of this problem with GAMS/CPLEX requires 0.15 CPU sec in HP-C160. The optimal objective function corresponds to 5771 units. The corresponding gantt chart is shown in Figure 3. Table 2 presents the results of the proposed formulation compared with the results found in the literature for this example. Note that the model of the proposed formulation involves less binary variables, 54 compared to 80 and 86(465) binary variables required by the other formulations. Also note that although the proposed formulation involves more constraints, the integrality gap is smaller which results in faster solution times and less number of nodes in the B&B tree. Note that there is a difference in the objective function obtained and those reported in the literature [8, 5]. This is due to the way in which the storage cost was considered in the objective function and the different time representation employed. The proposed formulation uses an exact continuous time representation while Sahinidis and Grossmann, [8], use discrete time formulation.

Conclusions

In this work, a new formulation for the short-term scheduling of batch plants was proposed. The formulation was tailored to accommodate intermediate due dates where specific product demands have to be satisfied. The mathematical model was based on the previous work of Ierapetritou and Floudas, [2, 3] where a new continuous time formulation was presented to effectively address the problem of short-term scheduling in batch, continuous and mixed production facilities where product demands are specified at the end of time horizon. Further exploitation of the problem special features results in the most efficient solution of the problem.

Acknowledgments

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	Pinto and Grossmann	Proposed Approach
binary vars	441	57
continuous vars	875	172
constraints	1791	559
CPU time	1257.17*	0.280
nodes	204	5
* HP 9000-730 workstation		

Table 1: Single Stage Results with Preordering - 29 orders

	Proposed Formulation	Sahinidis & Grossmann	Mockus & Reklaitis (NUCM) GOA	(UDM) B&B
Constraints	441	366	196	2438
Variables	316	326	218	1351
Binary vars	54	80	86	465
Integer optimum	5771	5593	-	-
Relaxation optimum	5861	5943	-	-
Integrality Gap	0.015	0.059	-	-
No. Nodes	10	2635	96	146
CPU Time (s)	0.15	97*	-	-
∗ IBM-3090 using MPSX-MIP/370				

Table 2: Results, Batch 2

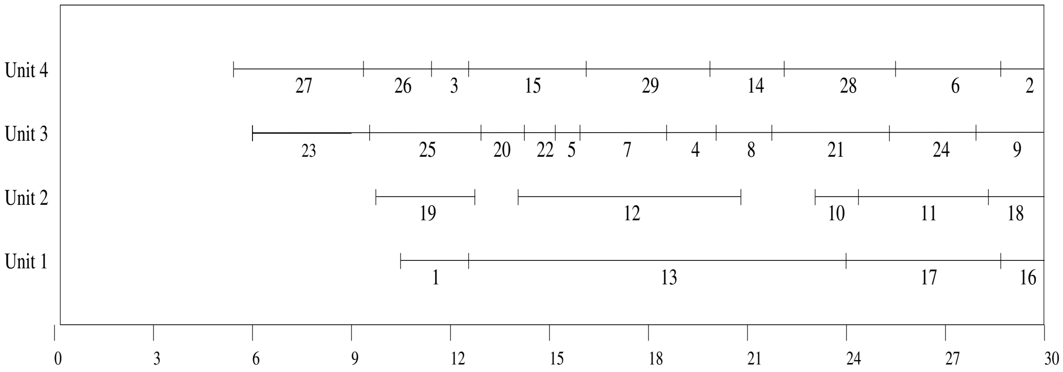

Figure 1: Gantt Chart for Preordered Single Stage problem: 29 orders

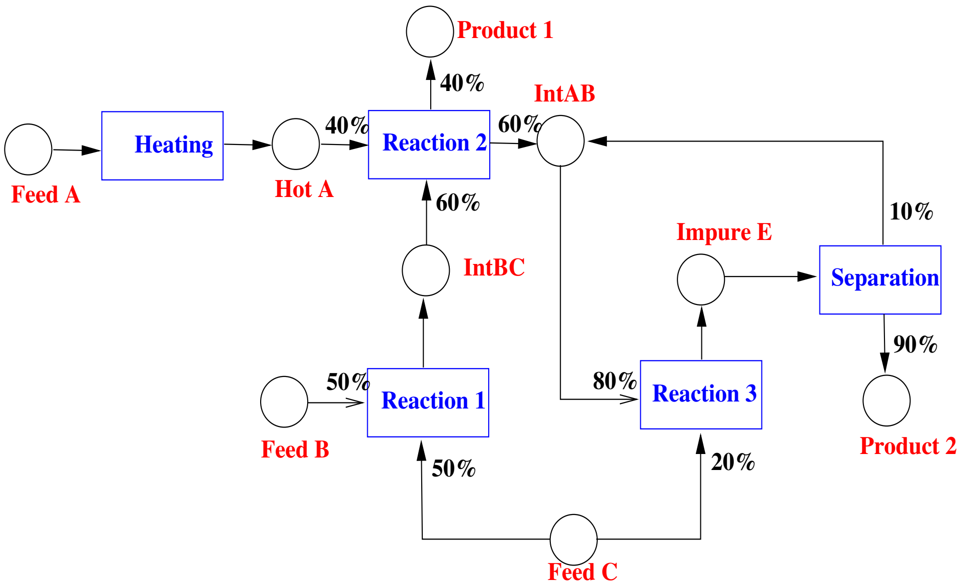

Figure 2: State Task Network for Batch 2

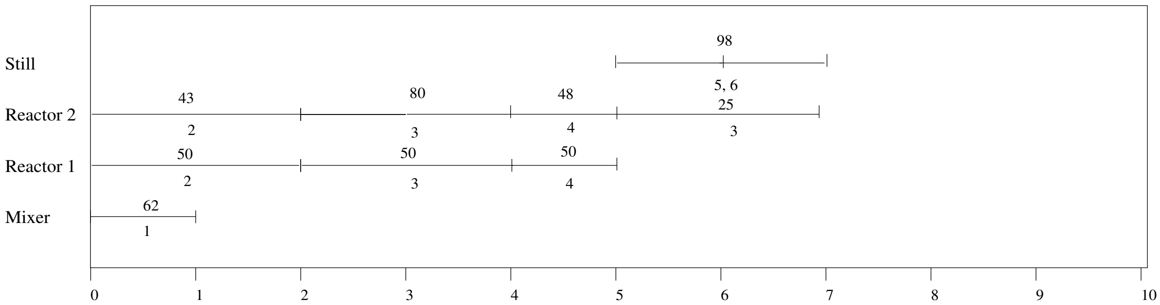

Figure 3: Gantt Chart for Batch 2